RAY OPTICS AND OPTICAL INSTRUMENTS

1. A ray is incident on a refracting surface of RI μ at an angle of incidence i and the corresponding angle of refraction is r. The deviation of the ray after refraction is given by $\delta = i - r$. Then, one may conclude that **[NSEP-2017]**

(A) r increases with i (B) δ increases with i

(C) δ decreases with i (D) the maximum value of δ is $\cos^{-1}\left(\frac{1}{\mu}\right)$

Ans. (A, B, D)

Sol. $\delta = i - r$ when $i\uparrow, r\uparrow, \delta\uparrow$

$$\delta_{max} = 90 - \sin^{-1}\frac{1}{u} = \cos^{-1}\frac{1}{u}$$

2. A convex lens and concave lens are kept in contact and the combination is used for the formation of image of a body by keeping it at different places on the principal axis. The image formed by this combination of lenses can be: **[NSEP-2017]**

(A) Magnified, inverted and real (B) Diminished, inverted and real

(C) Diminished, erect and virtual (D) Magnified, erect and virtual

Ans. (A, B, C, D)

Sol. Combination may behave converging

In that case (A), (B) & (D) are possible.

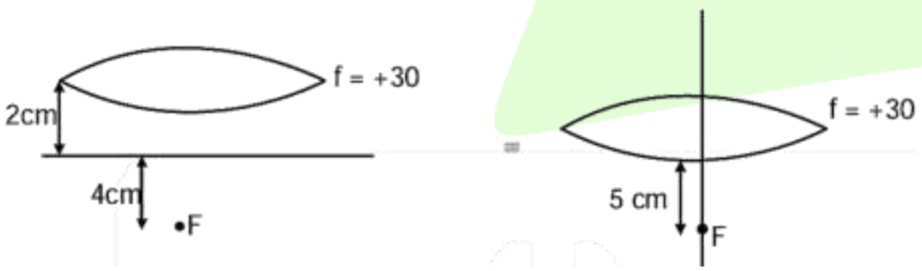
If combination behaves like diverging c will be correct. So all the options are correct.

3. A small fish, 4 cm below the surface of a lake, is viewed through a thin converging lens of focal length 30 cm held 2 cm above the water surface. Refractive index of water is 1.33. The image of the fish from the lens is at a distance of **[NSEP-2017]**

(A) 10 cm (B) 8 cm (C) 6 cm (D) 4 cm

Ans. (C)

Sol.



water $m = \frac{4}{3}$

$$\frac{1}{v} - \frac{1}{-5} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{6}{30}$$

$$\frac{1}{v} = -\frac{5}{30}$$

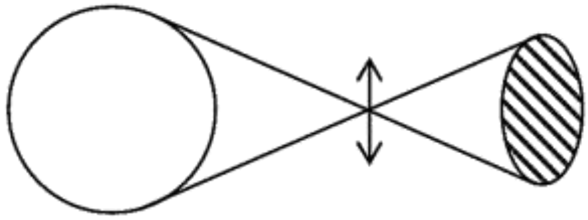
v = 6 cm

4. When observed from the earth the angular diameter of the sun is 0.5 degree. The diameter of the image of the sun when formed in a concave mirror of focal length 0.5 m will be about **[NSEP-2017]**

(A) 3.0 mm (B) 4.4 mm (C) 5.6 mm (D) 8.8 mm

Ans. (B)

Sol.



$$D_1 = \frac{5}{10} \times 0.5 \times \frac{\pi}{180} = \frac{\pi}{180 \times 4} = 4.36 \text{ mm} \approx 4.4 \text{ mm}$$

5. A point source of light is viewed through a plate of glass of thickness t and of refractive index 1.5. The source appears **[NSEP-2017]**

(A) closer by a distance $2t/3$ (B) closer by a distance $t/3$

(C) farther by a distance $t/3$ (D) farther by a distance $2t/3$

Ans. (B)

Sol. $\Delta y = t\left(1 - \frac{1}{\mu}\right) = t\left(1 - \frac{2}{3}\right) = \frac{t}{3}$ closer.

6. In cases of real images formed by a thin convex lens, the linear magnification is (I) directly proportional to the image distance, (II) inversely proportional to the object distance, (III) directly proportional to the distance of image from the nearest principal focus, (IV) inversely proportional to the distance of the object from the nearest principal focus. From these the correct statements are: **[NSEP-2017]**

(A) (I) and (II) only. (B) (III) and (IV) only

(C) (I), (II),(III) and (IV) all. (D) None of (I), (II), (III) and (IV).

Ans. (B)

Sol. $m = \frac{v}{u} \Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow 1 - \frac{v}{u} = \frac{v}{f} \Rightarrow 1 - m = \frac{v}{f}$

$m = 1 - \frac{v}{f} = \frac{f - v}{f}$

and $\frac{u}{v} - 1 = \frac{u}{f} \Rightarrow \frac{u}{v} = \frac{1 + u}{f} = \frac{f + u}{f}$

$m = \frac{v}{u} = \frac{f}{f + u}$

$u = -(f + x) \Rightarrow m = \frac{f}{f - f - x} = -\frac{f}{x}$

$v = f + y \Rightarrow m = \frac{f - f - y}{f} = -\frac{y}{f}$

$m \propto \frac{1}{x}, m \propto \frac{1}{y}$

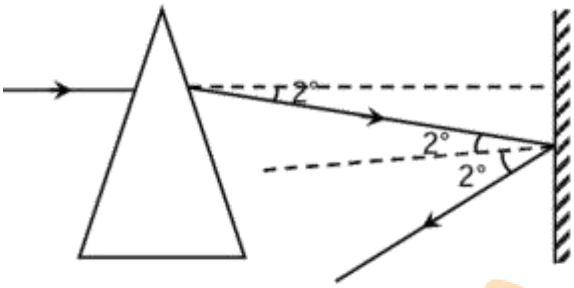
7. A horizontal ray of light passes through a prism of refractive index 1.5 and apex angle 4° and then strikes a vertical plane mirror placed to the right of the prism. If after reflection, the ray is to be horizontal, then the mirror must be rotated through an angle **[NSEP-2017]**

(A) 1° clockwise (B) 1° anticlockwise

(C) 2° clockwise (D) 2° anticlockwise

Ans. (A)

Sol.



$\delta = (1.5-1)4^{\circ} = 2^{\circ}$

Mirror must be rotated by 1° in clockwise direction (A)

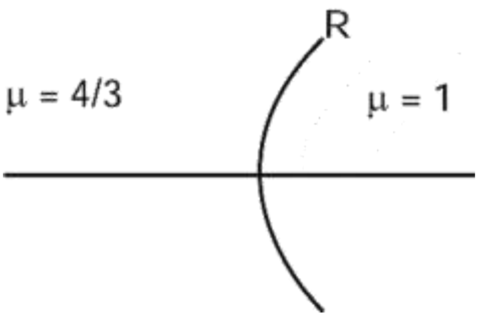
8. Rays from an object immersed in water ($\mu = 1.33$) traverse a spherical air bubble of radius R. If the object is located far away from the bubble, its image as seen by the observer located on the other side of the bubble will be **[NSEP-2017]**

(A) virtual, erect and diminished (B) real, inverted and magnified

(C) virtual, erect and magnified (D) real, inverted and diminished

Ans. (A)

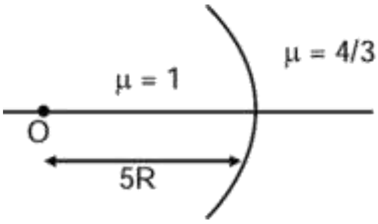
Sol. e-1



$$\frac{1}{v} - \frac{\frac{4}{3}}{-\infty} = \frac{1-\frac{4}{3}}{R} = -\frac{1}{3R} \Rightarrow v = -3R$$

$$m_1 = \frac{\frac{-3R}{1}}{\frac{\infty}{-\frac{4}{3}}} = +0$$

e-2:



$$\frac{4}{3v}-\frac{1}{-5R}=\frac{\frac{4}{3}-1}{-R}$$

$$\frac{4}{3v}+\frac{1}{5R}=-\frac{1}{3R}\Rightarrow\frac{4}{3v}=-\frac{1}{3R}-\frac{1}{5R}=-\frac{8}{15R}$$

$$v=-\frac{5R}{2}$$

$$m_2=\frac{-\frac{5R}{2(4/3)}}{-\frac{5R}{1}}=+\frac{3}{8}$$

$$m_1m_2\approx+0$$

image is virtual, erect and diminished.

9. A student uses a convex lens to determine the width of a slit. For this he fixes the positions of the object and the screen and moves the lens to get a real image on the screen. The images of the slit width are found to be 2.1 cm and 0.48 cm wide respectively when the lens is moved through 15 cm. Therefore, the slit width and the focal length of the lens respectively, are. **[NSEP-2017]**

(A) 1 cm, 9.3 cm (B) 1 cm, 10.5 cm (C) 2 cm, 12.8 cm (D) 2 cm, 15.2 cm

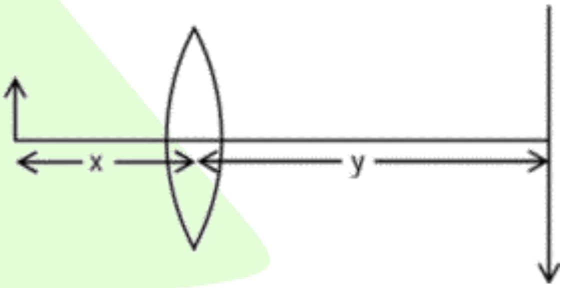
Ans. (A)

Sol. $h_0=\sqrt{h_1h_2}=\sqrt{2.1\times0.48}$

$=\sqrt{0.7\times3\times0.16\times3}$

$=3\times0.4\times\sqrt{0.7}$

$=1.2\times\sqrt{0.7}=1\text{ cm}$



For case I

$$\frac{h_i}{h_0}=\frac{2.1}{1}=2.1\text{ cm}=\frac{y}{x}$$

$y=2.1x\#(i)$

$y-x=15\text{ cm}$

$2.1x-x=15$

$1.1x=15$

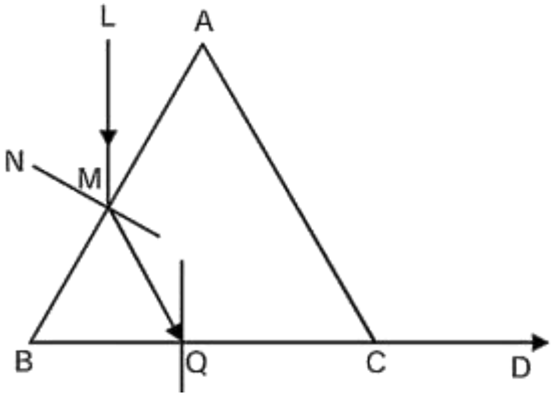
$$x=\frac{150}{11}\text{ cm}=13.6$$

$$\frac{1}{2.1x}-\frac{1}{-x}=\frac{1}{f}$$

$$\frac{2.1+1}{2.1x}=\frac{1}{f}$$

$$\Rightarrow f=\frac{2.1x}{3.1}=\frac{21}{31}\times\frac{150}{11}\text{ cm}\simeq9.3\text{ cm}$$

10. The following figure shows the section ABC of an equilateral triangular prism. A ray of light enters the prism along LM and emerges along QD. If the refractive index of the material of the prism is 1.6, angle LMN is **[NSEP-2017]**



(A) 35.6° (B) 37.4° (C) 39.4° (D) 41.3°

Ans. (A)

11. An object 1 cm long lies along the principal axis of a convex lens of focal length 15 cm, the centre of the object being at a distance of 20 cm from the lens. Therefore, the size of the image is **[NSEP-2017]**

(A) 0.3 cm (B) 3 cm (C) 9 cm (D) 12 cm

Ans. (C)

Sol. $u = -20 f = +15$

$$\frac{1}{v}+\frac{1}{20}=\frac{1}{15} \Rightarrow \frac{1}{v}=\frac{4}{60}-\frac{3}{60} \Rightarrow v = 60$$

$$\left|\frac{dv}{du}\right| = \left|\frac{v^2}{u^2}\right| = 9$$

$du = 1$ cm

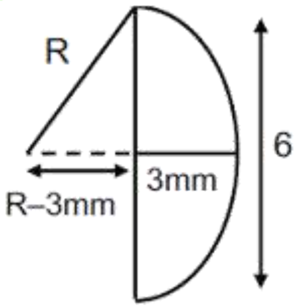
$dv = 9$

12. The aperture diameter of a plano-convex lens is 6 cm and its thickness is 3 mm. If the speed of light through its material is $v = 2\times10^8$ m / s, the focal length of the lens is **[NSEP-2018]**

(A) 40 cm (B) 35 cm (C) 30 cm (D) 20 cm

Ans. (C)

Sol. $\mu = 1.5$



Using Pythagoras theorem

$$\sqrt{3^2+(R-.3)^2} = R$$

$$9 + R^2 + (.3)^2 - .6R = R^2$$

$R = 15.15$

Using lens maker formula

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{R}\right)$$

$$= \frac{1}{2}\left(\frac{1}{15.15}\right)$$

$f = 30.3$ cm

13. The Hubble telescope in orbit above the earth has a 2.4 m circular aperture. The telescope has equipment for detecting ultraviolet light. The minimum angular separation between two objects that the telescope can resolve in ultraviolet light of wavelength 95 nm is **[NSEP-2018]**

(A) 4.83×10^{-8} rad (B) 4.03×10^{-8} rad

(C) 2.41×10^{-8} rad (D) 2.00×10^{-8} rad

Ans. (A)

Sol. 2.4 m circular aperture

$\lambda = 95$ nm

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 95 \times 10^{-9}}{2.4 \text{ m}} = 4.83 \times 10^{-8} \text{ radius}$$

14. The critical angle for light passing from glass to air is minimum for the light of wavelength **[NSEP-2018]**

(A) 0.7μ m (B) 0.6μ m (C) 0.5μ m (D) 0.4μ m

Ans. (D)

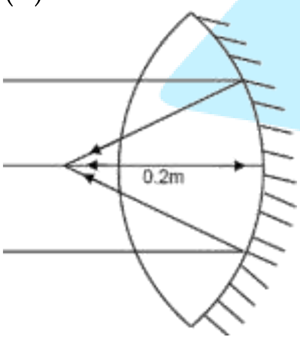
Sol. $\sin\theta_c = \frac{\mu_{air}}{\mu_{glass}} = \frac{1}{\mu_{glass}}$

$\mu = a + \frac{b}{\lambda^2} \Rightarrow \theta_c$ will be minimum for $\lambda = 0.4\mu$ m

15. A thin hollow equiconvex lens, silvered at the back, converges a beam of light parallel to the principal axis at a distance 0.2 m. When filled with water $\left(\mu = \frac{4}{3}\right)$, the same beam will be converged at a distance of **[NSEP-2018]**

(A) 0.40 m (B) 0.20 m (C) 0.12 m (D) None of the above

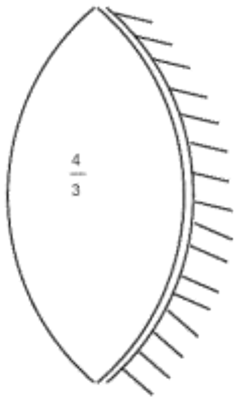
Ans. (C)



Sol.

$R = 0.4$ m $= 40$ cm

$$\frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_e}$$



$$=\frac{1}{-20}-2\times\left(\frac{4}{3}-1\right)\left(\frac{1}{40}-\left(\frac{-1}{40}\right)\right)$$

$$=\frac{-1}{20}-2\times\frac{1}{3}\times\frac{1}{20}$$

$$=-\frac{1}{20}-\frac{1}{30}=-\left(\frac{3+2}{60}\right)=\frac{-1}{12}$$

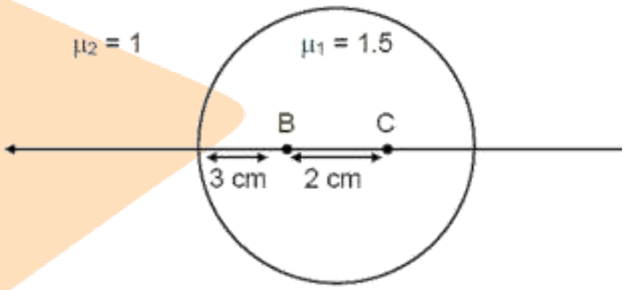
$$f_{eq}=12$$

16. An air bubble is situated at a distance 2.0 cm from the centre of a spherical glass paperweight of radius 5.0 cm and refractive index 1.5. The bubble is seen through the nearest surface. It appears at a distance v from the centre. Therefore, v is **[NSEP-2018]**

(A) 3.75 cm (B) 3.25 cm (C) 2.50 cm (D) 3.80 cm

Ans. (C)

Sol.



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1}{v}-\frac{1.5}{-3}=\frac{1-1.5}{-5}$$

$$\frac{1}{v}+\frac{1}{2}=\frac{1}{10}$$

$$\frac{1}{v}=\frac{1}{10}-\frac{1}{2}=\frac{1-5}{10}=\frac{-4}{10}=\frac{-2}{5}$$

$$v=\frac{-5}{2}=-2.5\text{ cm}$$

17. A concave lens of focal length f produces an image $(1/n)$ times the size of the object. The distance of the object from the lens is **[NSEP-2018]**

(A) $(n+1)f$ (B) $\frac{(n-1)}{n}f$ (C) $\frac{(n+1)}{n}f$ (D) $(d-1)f$

Ans. (D)

Sol. $$\frac{h_i}{h_0}=\frac{1}{n}=\frac{v}{u}=\frac{f}{u+f}$$

$$u+f=nf$$

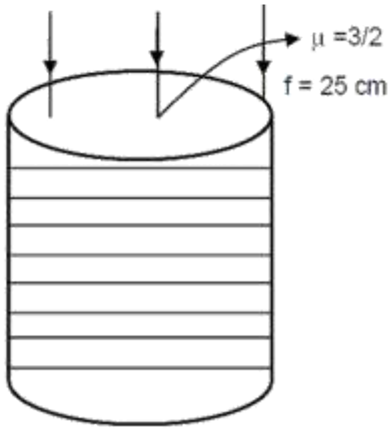
$$u=(n-1)f$$

18. A cylinder containing water (refractive index 4/3) is covered by an equiconvex glass (refractive index $3/2$) lens of focal length 25 cm. At the mid-day when the sun is just overhead, the image of the sun will be seen at a distance of **[NSEP-2018]**

(A) 100 cm. (B) 50 cm. (C) 37.5 cm. (D) 25 cm.

Ans. (B)

Sol. $\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{n_2-1}{R_1}+\frac{n_w-n_L}{R_2}$



$\frac{4}{3v}=\frac{\frac{3}{2}-1}{R_1}+\frac{\frac{4}{3}-\frac{3}{2}}{R_2}$

$\frac{1}{f}=\left(\frac{3}{2}-1\right)\frac{2}{R}$

$\frac{1}{25}=\frac{1}{R}$

$\frac{4}{3v}=\frac{1}{2(25)}+\frac{1}{6(25)}$

$\frac{4}{3v}=\frac{3}{150}+\frac{1}{150}$

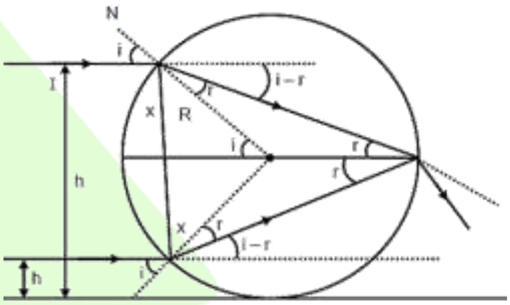
$v=50$ cm

19. A transparent cylindrical rod of length $\ell=50$ cm, radius $R=10$ cm and refractive index $\mu=\sqrt{3}$ lies onto a horizontal plane surface. A ray of light moving perpendicular to its length is incident on the rod horizontally at a height h above the plane surface such that this ray emerges out of the rod at a height 10 cm above the plane surface. Therefore, h is: **[NSEP-2018]**

(A) 1.34 cm. (B) 1.73 cm. (C) 10.0 cm. (D) 18.66 cm.

Ans. (A, C, D)

Sol. $r=i-r\Rightarrow r=\frac{i}{2}$



$1\sin i=\sqrt{3}\sin r$

$1\sin i=\sqrt{3}\frac{\sin i}{2}$

$\Rightarrow 2\sin(i/2)\cos(i/2)=\sqrt{3}\sin(i/2)$

$\Rightarrow \cos(i/2)=\frac{\sqrt{3}}{2}\Rightarrow i=60^\circ$

$\sin i=\frac{x}{R}\Rightarrow \sin 60^\circ=\frac{x}{10}$

$\Rightarrow x=\frac{10\sqrt{3}}{2}=8.66$ cm

Required $h=R+x$ 8 $h=R-x$

$=10+8.66$ cm $h=10-8.66$

$=18.66$ cm $=1.34$ cm

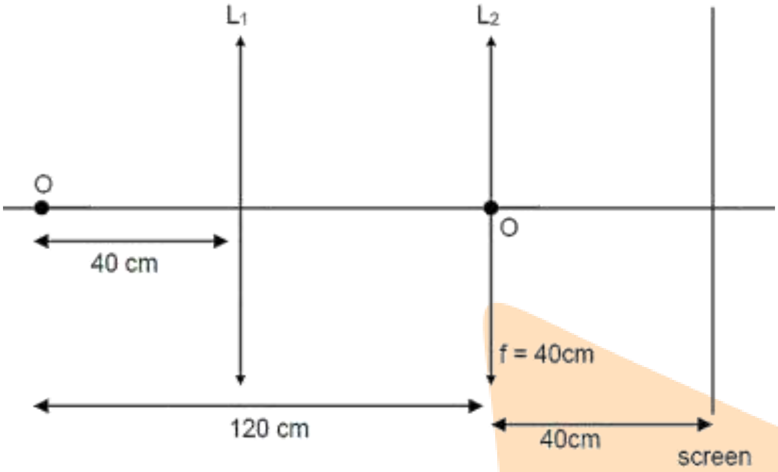
The correction ray will go undevided and among through the opposite end if it is incident at $h=10$ cm.

20. A converging lens of focal length 40 cm is fixed at 40 cm in front of a screen. An object placed 120 cm from the fixed lens is required to be focused on the screen by introducing another identical lens in between. The second lens should be placed at a distance x from the object where x is **[NSEP-2018]**

(A) 40 cm. (B) 50 cm. (C) 140 cm. (D) 150 cm.

Ans. (A, C)

Sol.



For L_2 to form image on screen

$v = 40$ cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{40}-\frac{1}{u}=\frac{1}{40} \Rightarrow u=-\infty$$

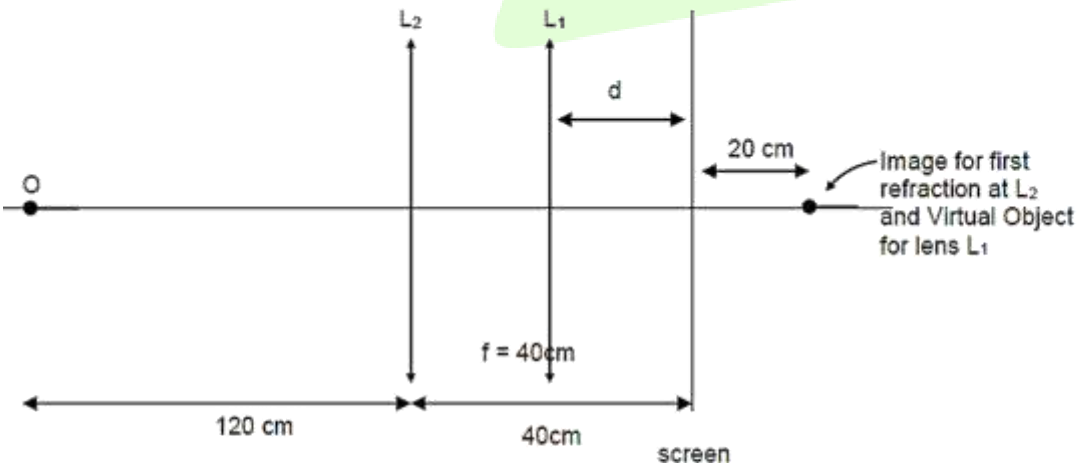
As the screen is in the focal plane of the lens L_2 the rays for image formation on the screen must be incident parallel on the lens. Therefore in order to create a parallel beam from the rays coming from the object the object must lie on the focal point of the other identical lens (call it L_1) $\therefore x = 40$ cm.

Another case

For lens to be b/w L_2 & screen. Refraction at L_2

$u=-120$ $f=40$

$$v=\frac{fu}{u+f}=\frac{40\times(-120)}{-80}=+60$$



Continued

for ref. at L_7

$u = d + 20 \quad v = d$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{d} - \frac{1}{d+20} = \frac{1}{40}$

$\Rightarrow \frac{d+20-d}{d^2+20d} = \frac{1}{40}$

$\Rightarrow d^2 + 20\,d - 800 = 0$

$\Rightarrow (d+40)(d-20) = 0$

$\Rightarrow d = 20$

$x = 160 - d = 160 - 20 = 140$

$\therefore$ (C) is also correct

21. A concave mirror has a radius of curvature R and forms the image of an object placed at a distance 1.5 R from the pole of the mirror. An opaque disc of diameter half the aperture of the mirror is placed with the pole at the centre. As a result **[NSEP-2019]**

(A) The position of the image will be the same but its central half will disappear

(B) The position of the image will be the same but its outer half will disappear

(C) The complete image will be seen at the same position and it will be exactly identical with the initial image

(D) The complete image will be seen at the same position but it will not be identical in all respect with the initial image.

Ans. (D)

Sol. Image will be formed at the same position but with lesser intensity because some rays will be blocked by the disc.

Correct answer should be D

22. A ray of white light is made incident on the refracting surface of a prism such that after refraction at this surface, the green component falls on the second surface at its critical angle. The colours present in the emergent beam will be **[NSEP-2019]**

(A) violet, indigo and blue.

(B) violet, indigo, blue, yellow, orange and red.

(C) yellow, orange and red.

(D) all colours

Ans. (C)

Sol. $\lambda \uparrow, n \downarrow, c \uparrow$

Max critical angle for red

$\rightarrow$ C increases

VIBG YOR

So Y, O & R will be present

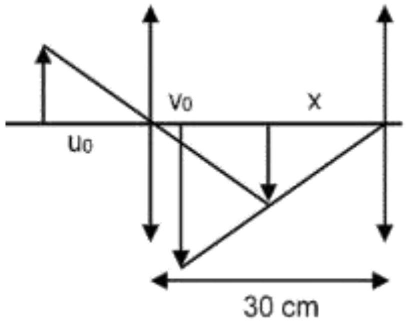
So Answer is (C)

23. In a compound microscope, having tube-length 30 cm , the power of the objective and the eyepiece are 100D and 10D respectively. Then the magnification produced by the microscope when the final image is at the least distance of distinct vision (25 cm) will be **[NSEP-2019]**

(A) 55 (B) 64 (C) 77 (D) 90

Ans. (C)

Sol.



$$x=\frac{Df_e}{D+f_e}$$

$$v_0+\frac{Df_e}{D+f_e}=30$$

$$v_0+\frac{25\times10}{35}=30$$

$$v_0=30-\frac{250}{35}$$

$$=30-\frac{50}{7}=\frac{160}{7}$$

$$\frac{7}{160}-\frac{1}{u_0}=\frac{1}{1}\Rightarrow\frac{1}{u_0}=\frac{7}{160}-1$$

$$=-\frac{153}{160}$$

$$u_0=-\frac{160}{153}$$

$$m=\frac{\frac{160}{7}}{\frac{160}{153}}\times\left\{1+\frac{25}{10}\right\}$$

$$=\frac{153}{7}\left\{\frac{35}{10}\right\}$$

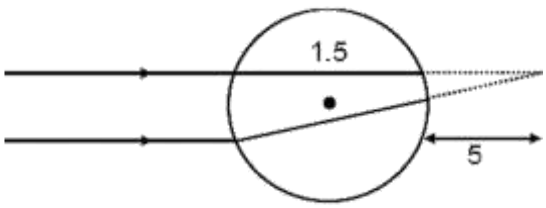
$$\frac{153}{2}=76.5$$

24. Parallel rays are incident on a glass sphere of diameter 10 cm and having refractive index 1.5. The sphere converges these rays at a certain point. The distance of this point from the centre of the sphere will be **[NSEP-2019]**

(A) 2.5 cm (B) 5 cm (C) 7.5 cm (D) 12.5 cm

Ans. (C)

Sol. First surface $\frac{1.5}{V}-\frac{1}{\infty}=\frac{1.5-1}{5}$



$$\Rightarrow\frac{V}{1.5}=10$$

$$\Rightarrow V=15$$

Second surface $\frac{1}{V_f}-\frac{3}{10}=\frac{1}{10}$

$$\Rightarrow\frac{1}{V_f}=\frac{4}{10}$$

$$\Rightarrow V_f=2.5$$

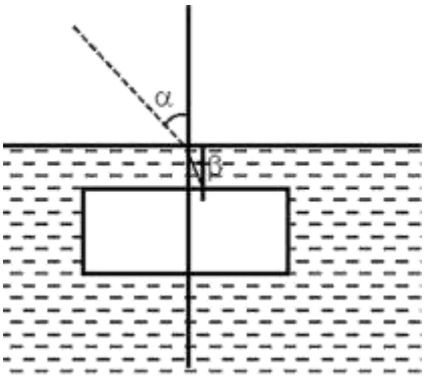
Hence distance from centre of sphere is 7.5 cm.

25. A rectangular slab of glass of refractive index 1.5 is immersed in water of refractive index 1.33 such that the top surface of the slab remains parallel to water level. Light from a point source in air is incident on the surface of water at an angle α such that the light reflected from the glass slab is plane polarised, the angle α is **[NSEP-2019]**

(A) 84.4° (B) 48.4° (C) 56.3° (D) 53.1°

Ans. (A)

Sol.



$$\sin\alpha = 1.33\sin\frac{1.33}{1.5}$$

$$\alpha = 84.4^{\circ}$$

26. A converging beam of light is pointing to P. Two observations are made with (i) a convex lens of focal length 20 cm and, (ii) a concave lens of focal length 16 cm placed in the path of the convergent beam at a distance 12 cm before the point P. It is observed that **[NSEP-2019]**

(A) In both cases the images are real.
(B) In both cases the images are virtual.
(C) For (i) the image is real and for (ii) the image is virtual.
(D) For (i) the image is virtual and for (ii) the image is real.

Ans. (A)

Sol. For convex lens

$u = +12$

$f = 20$

$$v = \frac{fu}{u+f} = \frac{20\times 12}{32} > 0 \Rightarrow \text{ Real image}$$

for concave lens

$u = +12$

$f = -16$

$$v = \frac{fu}{u+f} = \frac{(-16)(12)}{12-16} = \frac{-16\times 12}{-4} = 48 > 0 \Rightarrow 1 \text{ real image}$$

27. A pin of small length 'a' is placed along the axis of a concave mirror of focal length f, at the distance $u(>f)$ from its pole. The length of its image is 'b'. If the same object is placed perpendicular to its axis at the same distance u and the length of its image is now 'c', then. **[NSEP-2019]**

(A) $b = a\dfrac{f^2}{(u-f)^2}$ (B) $c = \sqrt{ab}$ (C) $c = b\dfrac{u-f}{f}$ (D) $bc = \dfrac{a^2f^3}{(u-f)^3}$

Ans. (A, B, C, D)

Sol. $m_L = \frac{b}{a} = m^2$

$m = \sqrt{\frac{b}{a}}$ and $m = \frac{f}{u-f}$

$\frac{h_1}{a} = \sqrt{\frac{b}{a}}$

$C = h_1 = \sqrt{ab}$

Also $m = \sqrt{\frac{b}{a}} = \frac{f}{u-f} \Rightarrow b = \frac{f^2 a}{(u-f)^2}$

$b = \left(\frac{f}{u-f}\right)^2 a$

$c = \left(\frac{f}{u-f}\right) a$

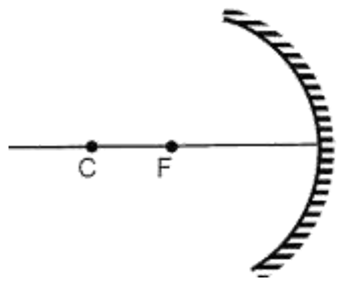
$\frac{b}{c} = \frac{f}{u-f}$

28. A thin rod of length 10 cm is placed along the axis of a concave mirror of focal length 30 cm in such a way that one end of the image coincides with one end of the object. The length of the image may be: **[NSEP-2019]**

(A) 7.5 cm (B) 12 cm (C) 15 cm (D) 10 cm

Ans. (A, C, D)

Sol. A and $A_1 \rightarrow$ same point at centre ($u = 60$ cm)



For B

$\frac{1}{V} + \frac{1}{-70} = \frac{1}{-30}$

$v = -52.5$

$L_1 = 7.5$ cm

Or $\frac{1}{v} + \frac{1}{-50} = \frac{1}{-30}$

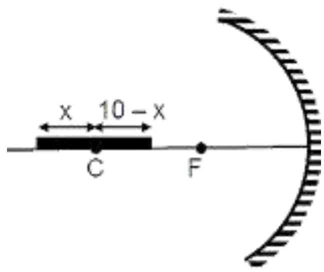
$v = -75$ cm

$L_1 = 15$ cm

For D

If the rod is kept such that one end is at C and rest of the rod is beyond C its image is formed with length shorter than the rod. Now if it is shifted towards the mirror eventually when the other end of the rod reaches C and the rod is between C and F its image is formed with length more than that of the rod. There fore in between these situations a case must arrive where the length of the image is same as the length of rod. It can be checked analytically as follows.

Suppose left end of the rod is placed at a distance x to the left of C and its image is being from on the other end of the rod.



Applying mirror formula

$\frac{1}{-(50+x)} + \frac{1}{-(60+x)} = \frac{1}{30}$

On solving

x = 5.413

29. A cylinder of length $\ell > 1$ filled with water $\left(\mu = \frac{4}{3}\right)$ up to the brim, kept on a horizontal table is covered at its top by an equiconvex glass ($\mu = 1.5$) lens of focal length 25 cm when in air. At mid day, 12.00 noon, Sun is just overhead and light rays comes parallel to the principal axis of the lens. The sun rays will be focused **[NSEP-2020]**

(A) 25 cm behind the lens in the water
(B) 37.5 cm behind the lens in the water
(C) 50 cm behind the lens in the water
(D) 100 cm behind the lens in the water

Ans. (C)

Sol. The focal length of a lens is obtained by $\frac{\mu_2}{f_2} = \frac{\mu - \mu_1}{R_1} - \frac{\mu - \mu_2}{R_2}$.... (1) where μ, μ_1 and μ_2 are the refractive indices of the material of lens, the object space and the image space respectively. R_1 and R_2 are the two radii of the lens. f_2 is the second focal length. When the lens is placed in air $\mu = 1.5, \mu_1 = 1$ and $\mu_2 = 1$ and then $\frac{1}{25} = \frac{1}{f} = \frac{1.5-1}{R} - \frac{1.5-1}{-R} \Rightarrow R = 25$ In the present case $\mu = 1.5, \mu_1 = 1$ and $\mu_2 = \frac{4}{3}$ Then equation (1) yields $\frac{4}{3f_2} = \frac{\frac{3}{2}-1}{25} - \frac{\frac{3}{2}-\frac{4}{3}}{-25} \Rightarrow f_2 = 50$ cm Hence the sun will be focused 50 below the lens.

30. A ray of light is incident on an equilateral prism made of flint glass (refractive index 1.6) placed in air. **[NSEP-2020]**

(A) The ray suffers a minimum deviation if it is incident at angle 53°.
(B) The minimum angle of deviation suffered by the ray is 46°.
(C) If prism is immersed in water $\left(\mu = \frac{4}{3}\right)$ the minimum deviation produced by the prism is 14°
(D) The minimum deviation produced by the prism is 23.6° if it is immersed in a liquid of refractive index $\mu = 1.2$

Ans. (A, B, C, D)

Sol. The refractive index of the prism is $\mu = 1.6 = \frac{\sin\left(\frac{A+\partial_m}{2}\right)}{\sin\frac{A}{2}}$ where ∂_m is the angle of minimum deviation. Using $\mu = 1.6$ and $A = 60^\circ$ One gets,

$\sin\frac{A+\partial_m}{2} = 0.8 \Rightarrow \frac{A+\partial_m}{2} = 53^\circ \Rightarrow \partial_m = 46^\circ$ Ans b also the angle of incidence here is

$i = \left(\frac{A+\partial_m}{2}\right) = \frac{60+46}{2} = 53^\circ$ Ans a

Now the prism is immersed in water of refractive index ${}_a\mu_w = \frac{4}{3}$, the angle of minimum deviation may now be obtained from ${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.6}{4/3} = 1.2 = \frac{\sin\left(\frac{A+\partial_m}{2}\right)}{\sin\frac{A}{2}} \Rightarrow \left(\frac{A+\partial_m}{2}\right) = 37^\circ \Rightarrow \partial_m = 14^\circ$A

When immersed in a liquid of refractive index ${}_a\mu_l = 1.2$,

The deviation may be obtained from ${}_l\mu_g \sin\frac{A}{2} = \sin\left(\frac{A+\partial_m}{2}\right)$

or $\frac{1.6}{1.2}\sin\frac{60}{2} = \sin\left(\frac{60+\partial_m}{2}\right) \Rightarrow \partial_m = 23.6^\circ$

31. The optical powers of the objective and the eyepiece of a compound microscope are 100 D and 20 D respectively. The microscope magnification being equal to 50 when the final image is formed at $d = 25$ cm i.e., the least distance of distinct vision. If the separation between the objective and the eyepiece is increased by 2 cm, the magnification of the microscope will be **[NSEP-2021]**

(A) 62 (B) 50 (C) 38 (D) 25

Ans. (A)

Sol. The magnifying power of a compound microscope when the final image is formed at D, the least distance of distinct vision is $MP = \frac{V_0}{U_0}\left(1+\frac{D}{f_e}\right)$ Now as per the given conditions $50 = \frac{V_0}{U_0}\left(1+\frac{25}{5}\right) \Rightarrow \frac{V_0}{U_0} = \frac{25}{3}$ Now for the objective lens $\frac{1}{V_0} - \frac{1}{-U_0} = \frac{1}{f_0} \Rightarrow 1 + \frac{V_0}{U_0} = \frac{V_0}{f_0}$ comparing the two, we get $V_0 = \frac{28}{3}$ cm. Increasing the length of microscope by 2 cm, Then $V_0 = \frac{28}{3} + 2 = \frac{34}{3}$ cm In the new situation $1 + \frac{V_0'}{U_0'} = \frac{V_0'}{f_0} = \frac{34}{3} \Rightarrow \frac{V_0'}{U_0'} = \frac{31}{3}$ The magnifying power therefore now becomes

$$MP = \frac{31}{3}\left(1+\frac{25}{5}\right) = 62$$

32. On a right angled transparent triangular prism ABC, when a ray of light is incident on face AB, parallel to the hypotenuse BC, it emerges out of the prism grazing along the surface AC. If instead the ray is made incident on face AC, parallel to the hypotenuse CB it gets totally reflected on face AB. The refractive index μ of the material of the prism is **[NSEP-2021]**

(A) $\mu > \sqrt{2}$ (B) $\sqrt{2} > \mu > \sqrt{\frac{3}{2}}$ (C) $\sqrt{3} > \mu > \sqrt{2}$ (D) $\mu < \sqrt{\frac{3}{2}}$

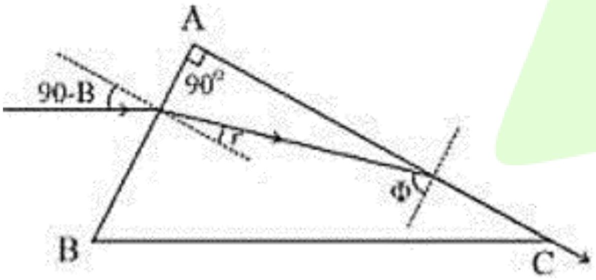
Ans. (B)

Sol. For a ray of light incident on side AB parallel to the base, we can write that the refractive index

$$\mu = \frac{\sin(90-B)}{\sin r} = \frac{\cos B}{\sin r} \text{ or } \mu \sin r = \cos B$$

Also $r + \phi = 90$ or $\sin r = \sin(90-\phi) = \cos\phi = \sqrt{1-\sin^2\phi}$

or $\sin r = \sqrt{1-\frac{1}{\mu^2}} = \sqrt{\frac{\mu^2-1}{\mu^2}}$ or $\mu \sin r = \sqrt{\mu^2-1}$



From (1) and (2) $\cos B = \sqrt{\mu^2-1}$

Or $\cos^2 B = (\mu^2-1)$

Next the ray is incident parallel to base on the side AC.

Here also $r_2 + r_3 = 90$ and since $r_3 > \phi$ i.e.the critical angle Now using $r_2 = 90 - r_3$ and

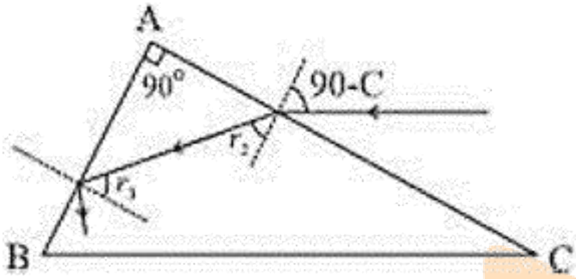
$$\mu = \frac{\sin(90-C)}{\sin r_2} = \frac{\sin(90-C)}{\sin(90-r_3)} = \frac{\cos C}{\cos r_3} \text{ or } \mu\cos r_3 = \cos C = \cos(90-B) = \sin B$$

This in turns gives $\mu^2\left(1-\sin^2 r_3\right)=\sin^2 B$ ____ (4) adding equation (3) and (4)

$$\left(\mu^2-\mu^2\sin^2 r_3\right)+\mu^2-1=1\Rightarrow 2\left(\frac{\mu^2-1}{\mu^2}\right)=\sin^2 r_3\,\#(5)$$

Further angle $r_3>\phi$ or $\sin r_3>\sin\phi$

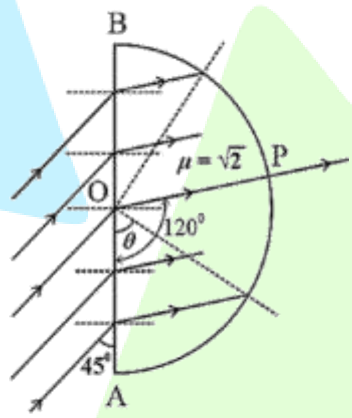
$$\Rightarrow\sqrt{2\left(\frac{\mu^2-1}{\mu^2}\right)}>\sin\phi\Rightarrow 2\left(\frac{\mu^2-1}{\mu^2}\right)>\frac{1}{\mu^2}\Rightarrow\mu^2-1>\frac{1}{2}\Rightarrow\mu^2>\frac{3}{2}\Rightarrow\mu>\sqrt{\frac{3}{2}}$$



Also $r+\phi=90$ but $r<\phi$ So essentially $\phi>45$ or $\sin\phi>\sin 45$ or $\frac{1}{\mu}>\frac{1}{\sqrt{2}}\Rightarrow\mu<\sqrt{2}$ thus we conclude that $\sqrt{\frac{3}{2}}<\mu<\sqrt{2}$

33. A parallel beam of light is made incident (as shown) on the flat diametric plane of a transparent semi-circular thin sheet of thickness $t(t\ll R)$ of refractive index $\mu=\sqrt{2}$ at an angle of 45°. As a result of refraction, the light enters the semi-circular sheet and comes out at its curved surface.

[NSEP-2021]



(A) Light rays come out at the curved surface for values of θ in the range $75^\circ\le\theta\le 165^\circ$.
(B) The range of angle θ is independent of the angle of incidence.
(C) The range of angle θ depends on the refractive index of the material
(D) All the emergent rays of light shall cross the line OP which is a refracted ray at $\theta=120^\circ$.
Here θ is the angle between the vertical diameter AB and the concerned radius of the semicircular sheet of radius R.

Ans. (A, C, D)

Sol. Snell's law is $\mu=\frac{\sin i}{\sin r}\Rightarrow\sqrt{2}=\frac{\sin 45}{\sin r}\Rightarrow\sin r=\frac{1}{2}\Rightarrow r=30^\circ$

The critical angle is $\sin^{-1}\frac{1}{\sqrt{2}}=45^\circ$.

For the emergence, the angle of incidence at curved surface must be less than 45° therefore angle θ should be greater than

$$\theta_{min} = 180 - \left(90 - \sin^{-1}\left(\frac{\sin i}{\mu}\right)\right) - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\theta_{min} = \left[180 - (90 - 30) - 45\right] = 75^\circ$$

Towards the upper edge angle θ must be less than

$$\theta_{max} = \left(90 + \sin^{-1}\left(\frac{\sin i}{\mu}\right)\right) + \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\theta_{max} = (90 + 30) + 45 = 165$$

Thus we obtain $\theta_{min} < \theta < \theta_{max}$ as $75^\circ \le \theta \le 165^\circ$ for the emergence of light through the curved surface. Thus the light will come out only for the angle θ lying within the range $\theta_{max} - \theta_{min} = 2\sin^{-1}\left(\frac{1}{\mu}\right)$ which is independent of the angle of incidence but depends on the refractive index (μ) of the material. Of course the range is same for all values of angle of incidence yet the values of θ_{min} and θ_{max} are different for different values of angle of incidence. Hence option b is not correct. The light coming out of the curved surface will go away from normal hence towards the line which is the increased radius for $\theta = 120^\circ$ and thus form a convergent beam towards the enhanced radius corresponding to $\theta = 120^\circ$.

34. A thin double convex lens of radii of curvature $R_1 = 20$ cm and $R_2 = 60$ cm is made-up of a transparent material of refractive index $\mu = 1.5$. Choose the correct option(s) **[NSEP-2021]**

(A) The focal length of the lens is $f = 30$ cm when in air.

(B) The lens behaves as a concave mirror of focal length $f_M = 10$ cm when silvered on the surface of radius $R_2 = 60$ cm

(C) The lens behave as a concave lens (diverging lens) if the image space beyond $R_2 = 60$ cm radius surface is filled with a transparent liquid of refractive index $\mu = \frac{5}{3}$. The object space prior to the surface of radius $R_1 = 20$ cm is air.

(D) A beam of rays incident parallel to principal axis focuses at 48 cm behind the lens if water $\left(\mu = \frac{4}{3}\right)$ fills the entire space behind the surface of radius $R_2 = 60$ cm. The object space prior to the surface of radius $R_1 = 20$ cm is air.

Ans. (A, B, D)

Sol. The focal length f2of a lens of refractive index μ and radii of curvature R_1 and R_2 when the refractive index of the object space is μ_1 and that of image space is μ_2 is calculated by $\frac{\mu_2}{f_2} = \frac{\mu - \mu_1}{R_1} - \frac{\mu - \mu_2}{R_2}$ If the lens is kept in air $\mu_1 = 1$ and $\mu_2 = 1$ then

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.5-1)\left(\frac{1}{20}-\frac{1}{-60}\right)=\frac{1}{30}\Rightarrow f=30\text{ cm hence a is correct}$$

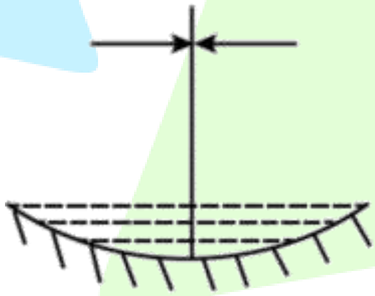
When the lens is silvered on the surface of radius 60 cm, it will behave as a concave mirror of focal length f_M such that $\frac{1}{f_M}=\Sigma\frac{1}{f}=\frac{1}{f_{lens}}+\frac{2}{R}+\frac{1}{f_{lens}}=\frac{1}{30}+\frac{2}{60}+\frac{1}{30}=\frac{6}{60}=\frac{1}{10}$ means $f_M=10$ cm

Hence the option b is correct.

When the image space is filled with a liquid of refractive index $\mu_2=\frac{5}{3}$, the object space still being air $(\mu_1=1)$, the second focal length of the lens is obtained by $\frac{5}{3f_2}=\frac{1.5-1}{+20}-\frac{1.5-\frac{5}{3}}{-60}\Rightarrow f_2=+75$ cm

Also the first focal length in this case is $f_1=+45$ cm so the lens still behaves as convex lens and not a concave (diverging) lens. Hence option c is wrong. Considering a different situation when air in object space and water $\left(\mu=\frac{4}{3}\right)$ in image space, the second focal length of lens then is $\frac{4}{3f_2}=\frac{1.5-1}{20}-\frac{1.5-4/3}{-60}=\frac{1}{36}\Rightarrow f_2=+48$ cm Hence a beam of light incident parallel to the principal axis focuses 48 cm behind the lens. Hence option d is correct.

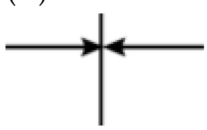
35. A concave mirror when place in air has a focal length $f=20$ cm. The mirror is now placed horizontally and filled with a thin layer of water having refractive index $\frac{4}{3}$. The object is placed horizontally near the principal axis at a distance d from the mirror such that a real, inverted image is formed at the same plane as the object, as shown in the figure. What is the value of d? **[NSEP-2022]**



(A) 30 cm (B) 20 cm (C) 15 cm (D) 40 cm

Ans. (A)

Sol.



$$\frac{1}{F_{eq}}=\frac{1}{F_m}-\frac{2}{F_l}\ldots(1)$$

Also, $\frac{1}{F_l}=\left(\frac{4}{3}-1\right)\left(\frac{1}{\infty}-\frac{1}{-40}\right)$

$\therefore F_l=+120$ cm

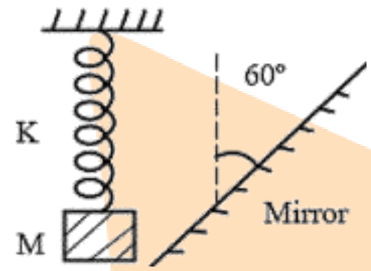
$\therefore$ from eqn. (1) $\Rightarrow \frac{1}{F_{eq}} = \frac{-1}{20} - \frac{2}{120}$

$\therefore$ The combination acts as a mirror of focal length, $F_{eq} = -15$ cm

$\therefore d = 2F \Rightarrow d = 30$ cm

$\rightarrow$ option (A)

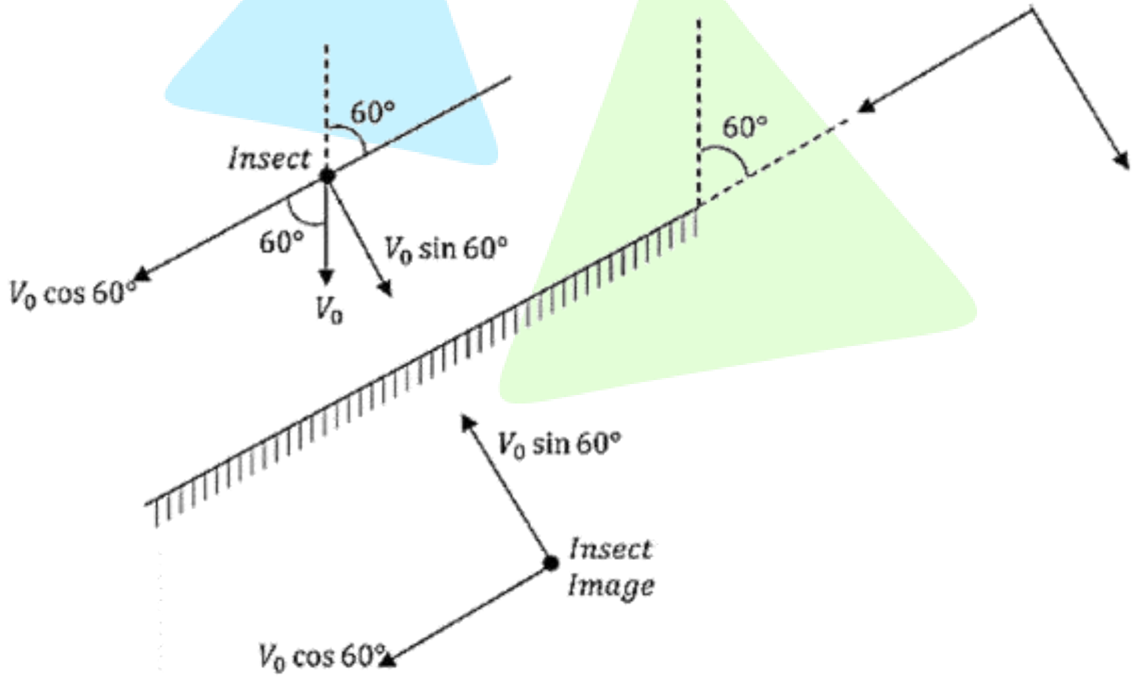
36. An insect of negligible mass is sitting on a block of mass M , tied with a spring of force constant K. The block performs simple harmonic motion vertically with amplitude A in front of a mirror which is include at 60° with the vartical as shown. The maximum speed of insect relative to its image will be **[NSEP-2022]**



(A) $2A\sqrt{\frac{K}{M}}$ (B) $A\sqrt{\frac{3K}{M}}$ (C) $A\sqrt{\frac{K}{M}}$ (D) Zero

Ans. (B)

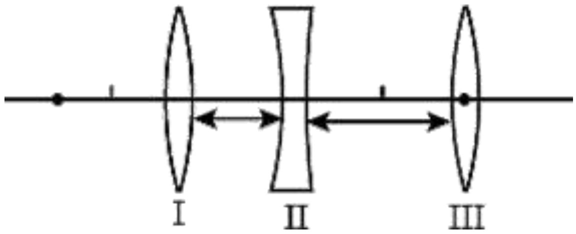
Sol. $\Rightarrow V_{max}$ of insect $= \omega A = \sqrt{\frac{K}{M}}A = V_0$



$\Rightarrow$ at $V_{max} \Rightarrow$

$\Rightarrow V_{rel} = V_0\sqrt{3} = \sqrt{3}\sqrt{\frac{K}{M}}A = \sqrt{\frac{3K}{M}}A$

37. A concave lens of focal length 10 cm is placed between two convex lenses of focal length 10 cm and 20 cm at a separation of 5 cm between the first and second lens and 10 cm between the second and third lens. An object is placed at 30 cm in front of the first convex lens. The final image is formed beyond the third lens at a distance v from it. Then **[NSEP-2022]**



(A) $v = 15$ cm (B) $v = \infty$ (C) $v = 45$ cm (D) $v = 20$ cm

Ans. (D)

Sol. For 1^{st} lens $\Rightarrow \frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$

$\Rightarrow v_1 = 15$ cm

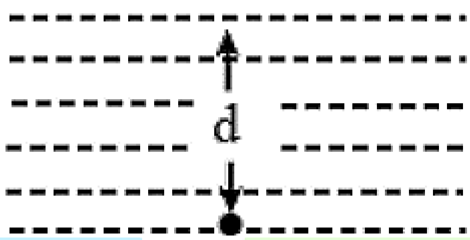
For 2^{nd} lens $\Rightarrow$ object distance $= (15-5) = +10$ cm (virtual object)

$\frac{1}{v_2} - \frac{1}{10} = \frac{1}{-10} \Rightarrow v_2 \to \infty \Rightarrow$ refracted rays are parallel

For 3^{rd} lens $\Rightarrow$ Parallel rays converge at focus

$\Rightarrow v = 20$ cm

38. A point source S of light is placed at a depth d below the surface of water in a large and deep lake. Fraction of light that escapes in space above directly from water (refractive index $= \mu$) surface is given by **[NSEP-2022]**



(A) $\sqrt{1-\frac{1}{\mu^2}}$

(B) $\frac{1}{2}\sqrt{1-\frac{1}{\mu^2}}$

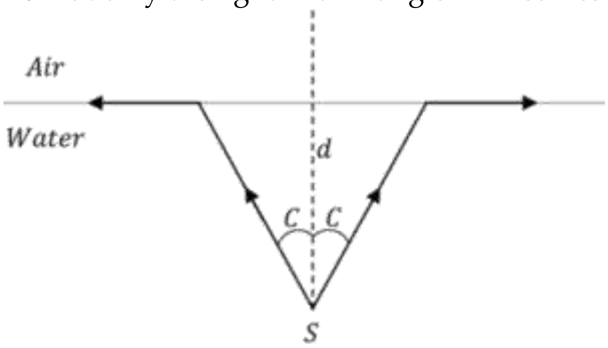
(C) $\frac{1}{2}\left\{1-\sqrt{1-\frac{1}{\mu^2}}\right\}$

(D) Depends on d and increases with increasing d

Ans. (C)

Sol. $\to \sin C = \frac{1}{\mu} \Rightarrow \cos C = \sqrt{1-\frac{1}{\mu^2}}$ $\to$ Point source emits light in all the directions.

$\Rightarrow$ But only the light within angle ' C ' comes out.



For a cone with semi-vertical angle Q, its solid angel is, $2\pi(1-\cos\theta)$

$$\Rightarrow \text{So, friaction } = \frac{2\pi(1-\cos\cos C)}{4\pi}$$

$$\Rightarrow \frac{1-\cos\cos C}{2} = \frac{1}{2}\left[1-\sqrt{1-\frac{1}{\mu^2}}\right]$$

39. A convex lens is held 45 cm above the bottom of an empty tank. The image of a point object at bottom of tank is formed 36 cm above the lens. Now a liquid is poured into the tank upto a height of 40 cm above the bottom. It is found that distance of image of same point object at the bottom of the tank is 60 cm above the lens. Refractive index of liquid is **[NSEP-2022]**

(A) 1.33 (B) 1.37 (C) 1.40 (D) 1.60

Ans. (D)

Sol. Case $1 \rightarrow$ without liquid in the tank, $v = 36 \,\&\, u = 45$ cm

Using Lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{36} - \frac{1}{45} = \frac{1}{f}$$

$\Rightarrow f = 20$

Case $2 \rightarrow$ with liquid in the tank upto a height 40 cm

$\Rightarrow$ Apparent depth from surface of liquid $\rightarrow d = \frac{40}{\mu}$

$\Rightarrow v = 60$ cm & object distance $(u) = -(5+d)$cm

Using Lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\rightarrow \frac{1}{60} - \frac{1}{-(5+d)} = \frac{1}{20}$$

$\Rightarrow d = 25$ cm

$$\Rightarrow d = \frac{40}{\mu} 25 \Rightarrow \mu = 1.6$$

40. Given that the critical angle of incidence for total internal reflection within a transparent material when placed in air is 45° The Brewster's angle of incidence for light propagating from air to the transparent material will be **[NSEP-2022]**

(A) 54.74° (B) 35.26° (C) 25.26° (D) 44.74°

Ans. (A)

Sol. Critical angle is given by, $\sin i_c = \frac{1}{\mu}$

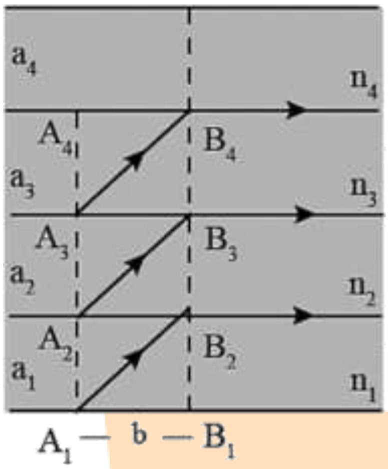
$$\Rightarrow \mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45^\circ}$$

$\Rightarrow \mu = \sqrt{2}$

Brewster's angle is given by, $\theta = \mu$

$\Rightarrow \theta = \tan^{-1}\sqrt{2} > 45^\circ$

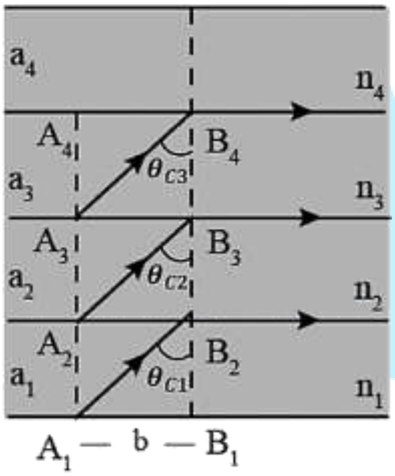
41. There are four layers of glass plates, placed on top of each other such that bottom one has thickness a_1 and refractive index $n_1 = 2.7$. Next one has thickness a_2 and refractive index $n_2 = 2.43$. The third one and the top one has thickness a_3 and a_4 and refractive indices n_3 and n_4 respectively. Three rays starting at the same moment from A_1, A_2 and A_3 reach points B_2, B_3, B_4 at the same time, with their angles of incidence being critical angle. You are given $A_1B_1 = A_2B_2 = A_3B_3 = A_4B_4 = b = 10$ mm. Choose correct statement (s). **[NSEP-2022]**



(A) $n_3 = 1.968$ (B) $n_4 = 1.291$ (C) $a_2 = 7.243$ mm (D) $a_3 = 11.51$ mm

Ans. (A, B, C, D)

Sol.



In first interface between n_1 and n_2,

$$\sin\theta_{c1} = \frac{n_2}{n_1} = \frac{2.43}{2.7} = \frac{9}{10}$$

$$\Rightarrow \tan\theta_{c1} = \frac{9}{\sqrt{19}} = \frac{b}{a_1} \Rightarrow a_1 = \frac{10\sqrt{19}}{9} \text{ mm}$$

Time taken for the light from A_1 to B_2 is given by,

$$t = \frac{A_1B_2}{v_1} = \frac{\sqrt{a_1^2 + b^2}}{\frac{c}{n_1}}$$

Time taken for the light from A_2 to B_3 is given by,

$$t = \frac{A_2B_3}{v_2} = \frac{\sqrt{a_2^2 + b^2}}{\frac{c}{n_2}}$$

Given that, the time taken is same in all the three cases,

$$\Rightarrow \frac{\sqrt{a_1^2+b^2}}{\frac{c}{n_1}} = \frac{\sqrt{a_2^2+b^2}}{\frac{c}{n_2}} \Rightarrow a_2 = 7.243 \text{ mm}$$

In second interface between n_2 and n_3,

$$\sin\theta_{c2} = \frac{n_3}{n_2}$$

Also, $\tan\theta_{c2} = \frac{b}{a_2}$

$\Rightarrow n_3 = 1.968$

Similarly, considering the time taken from A_2 to B_3 and A_3 to B_4

$$\Rightarrow\Rightarrow \frac{\sqrt{a_2^2+b^2}}{\frac{c}{n_2}} = \frac{\sqrt{a_3^2+b^2}}{\frac{c}{n_3}}$$

$\Rightarrow a_2 11.51$ mm

In third interface between n_3 and n_4, $\sin\theta_{c3} = \frac{n_4}{n_3}$

Also, $\tan\theta_{c3} = \frac{b}{a_3}$

$\Rightarrow n_4 = 1.291$

Correct option (ABCD)

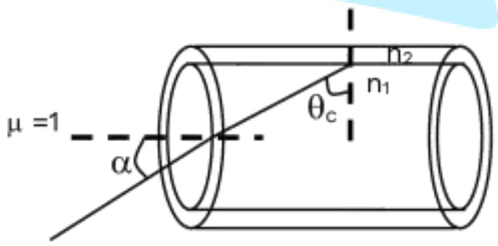
42. Numerical aperture of an optical fibre is a measure of **[NSEP-2023]**

(A) the attenuation of light through it (B) its resolving power

(C) the pulse dispersion through it (D) its light gathering power

Ans. (D)

Sol.



$\sin\alpha = \sqrt{n_1^2 - n_2^2}$

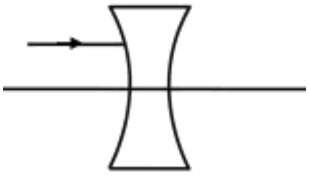
43. An equi-concave lens of radii of curvature of the two surfaces numerically equal to 7 cm and refractive index $\mu = 1.5$ has a small silver dot on the rear surface. As a result of this, a ray of light incident parallel to the principal axis gets reflected from its rear surface and then reflected also from the inner front surface. The ray after the second reflection emerges out of the thin lens and appears to focus at a point P on the principal axis. The point P lies **[NSEP-2023]**

(A) 1 cm before the lens (B) 2 cm before the lens

(C) 1 cm beyond the lens (D) at none of these

Ans. (A)

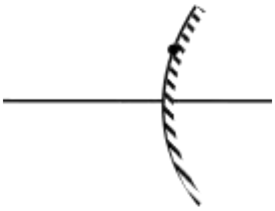
Sol.



$$\frac{1.5}{V}-\frac{1}{\infty}=\frac{1.5-1}{7}$$

$$\frac{3}{2V}=\frac{-1}{14}$$

$$V=-21$$



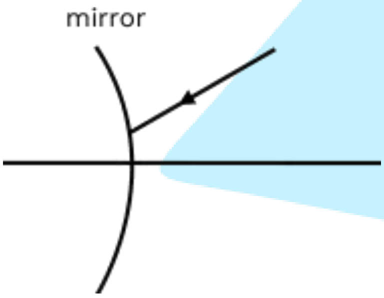
$$\frac{1}{V}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{V}+\frac{1}{-21}=\frac{2}{7}$$

$$\frac{1}{V}=\frac{1}{21}+\frac{2}{7}$$

$$=\frac{1+6}{21}=\frac{1}{3}$$

$$V=+3$$



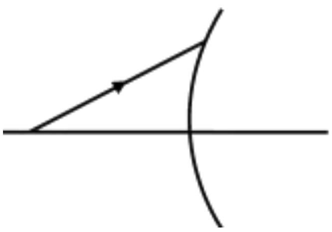
$$\frac{1}{V}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{V}+\frac{1}{-3}=\frac{2}{7}$$

$$\frac{1}{V}=\frac{2}{7}+\frac{1}{3}$$

$$\frac{1}{V}=\frac{13}{21}$$

$$V=\frac{21}{13}$$



$$\frac{1}{V}-\frac{3\times 13}{2(-21)}=\frac{1-1.5}{+7}$$

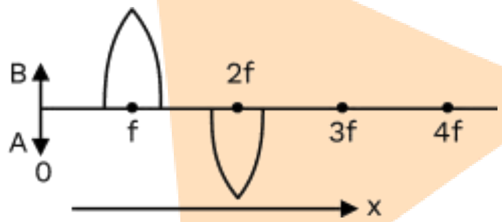
$$\frac{1}{V}+\frac{39}{42}=\frac{-1}{14}$$

$$\frac{1}{V}=-\frac{1}{14}\times\frac{42}{3}=-1$$

$$\frac{1}{V}=-\frac{1}{14}\left[1+\frac{39}{3}\right]$$

$$\frac{1}{V}=-\frac{1}{14}\times\frac{42}{3}=-1$$

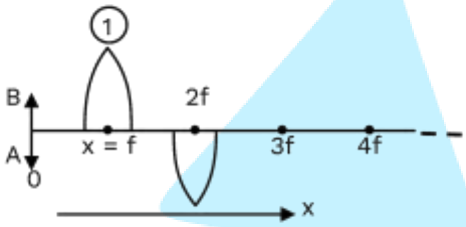
44. An equi-convex lens of focal length' f ' is cut along a diameter, in two halves (pieces). The two identical pieces of the lens are now arranged as shown in the figure on a common axis at a separation f between the two. The image of an object AB placed at $x=0$ cannot be formed at the distance $x=\xi$ from the object along the axis, for the value of ξ as **[NSEP-2023]**



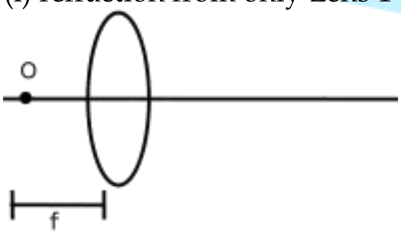
(A) $\xi=2f$ (B) $\xi=3f$ (C) $\xi=4f$ (D) $\xi=\infty$

Ans. (A)

Sol.



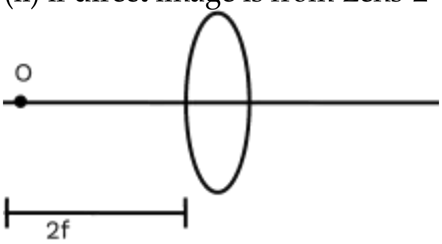
(i) refraction from only Lens 1 Then



$u=f$

$v=+\infty$

(ii) if direct image is from Lens-2

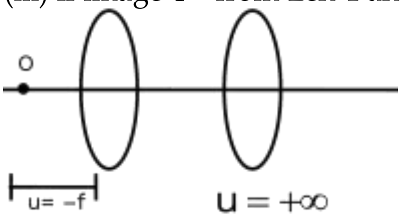


$u=-2f$

$v=-2f$

so $x=4f$

(iii) If image I^{st} from Len-1 and then from lens-2



$x = 3f$

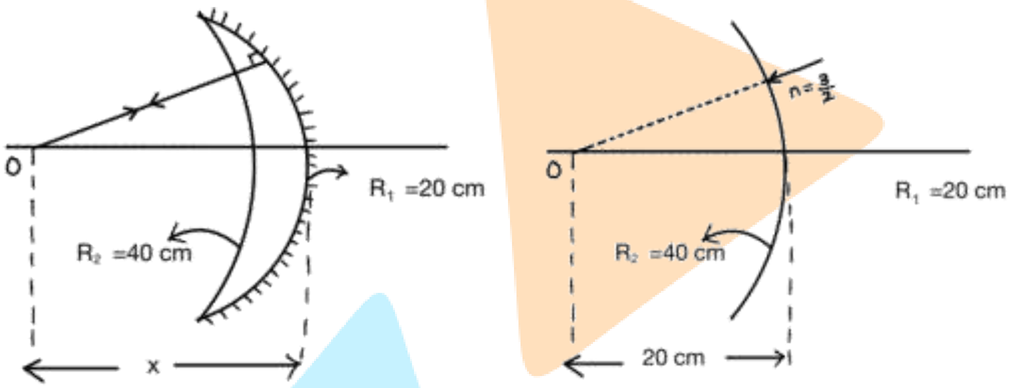
Ans will be $x = 2f$

45. The convex surface of a concavo-convex lens of refractive index 1.5 and radii of curvature $R_1 = 20$ cm and $R_2 = 40$ cm has been silvered so as to make it reflecting. The distance of a luminous object from the reflecting system when placed in front of it on its principal axis, so that the image coincides with the object is **[NSEP-2023]**

(A) 40 cm (B) 32 cm (C) 16 cm (D) 8 cm

Ans. (C)

Sol.



Say x is the distance of object from the combination so that image forms on itself.

A ray coming from object O, must strike the silenced surface perpendicularly as shown in figure. Therefore for other surface, object is virtual & is at distance 20 cm form the surface, thus for this surface.

$u = +20$ cm

$n = \frac{3}{2}$

$R = +40$ cm

$$\frac{1}{v} - \frac{3}{2(20)} = \frac{1-3/2}{40}$$

$$\frac{1}{v} = \frac{3}{40} - \frac{1}{80} = \frac{5}{80}$$

$v = 16$ cm

46. A direct vision spectroscope has been designed to obtain dispersion without deviation by arranging alternate inverted thin prisms of crown glass (refractive index $\mu_1 = \sqrt{2}$) and flint glass $\left(\mu_2 = \sqrt{3}\right)$ d with refracting angle $\theta_{flint} = 3^\circ$. The refracting angle θ_{crown} of the crown glass prism is

[NSEP-2023]

(A) 3.0° (B) 4.5° (C) 5.3° (D) 6.0°

Ans. (C)

Sol. Condition of dispersion without deviation is

$$(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$$

$$\left(\sqrt{2}-1\right)A_1 = \left(\sqrt{3}-1\right)3$$

$$A_1 = \left(\frac{\sqrt{3}-1}{\sqrt{2}-1}\right)\times 3$$

$$A_1 = \left(\sqrt{3}-1\right)\left(\sqrt{2}+1\right)\times 3 = 5.3^\circ$$

47. While performing an experiment for determining the focal length of a concave mirror by $u-v$ method, a student recorded the given sets of the positions (in cm) of the object and the corresponding image on the bench as $(12,51),(18,54),(30,50),(48,34),(42,42)$ and $(78,98)$. She used an optical bench of length 1.5 m and the mirror is fixed at the 90 cm mark on the bench. The maximum acceptable error in the location of the image is 0.2 cm. The reading (observation) that cannot be obtained from experimental measurement and has been incorrectly recorded, for a mirror of focal length $=24$ cm , is **[NSEP-2023]**

(A) $(18,54)$ (B) $(30,50)$ (C) $(48,34)$ (D) $(78,98)$

Ans. (D)

Sol. $u = -[x_L - x_0]$ & $V = -[x_L - x_i]$ & $f = \dfrac{uv}{u+v}$

for $(18,54)$

$u = -(90-18) = -72$ cm

$v = -(90-54) = -36$ cm

$$f = \frac{-72\times -36}{-72-36} = \frac{72\times 36}{36\times 3} = 24 \text{ cm}$$

for $(30,50)$

$$f = \frac{(90-30)(90-50)}{(90-30)+(90-50)}$$

$$\frac{60\times 40}{100} = 24 \text{ cm same as given}$$

for $(48,34)$

$$f = \frac{42\times 56}{42+56} = \frac{42\times 56}{98} = 24 \text{ cm}$$

48. Two identical transparent solid cylinders, each of radius 10 cm and refractive index $\mu = \sqrt{3}$, lie horizontally parallel to each other on a horizontal plane mirror with a separation x between their horizontal axes. A ray of light is incident horizontally on the cylinder A at a height h above the plane mirror so as to emerge from this cylinder at a height $h_1 = 0.1$ m above the plane mirror.
The ray emerging out from the first cylinder A is reflected from the horizontal plane mirror to enter the second parallel cylinder B at a height h_2 and then this ray emerges out of the second cylinder, parallel and in-line with the original incident ray. The correct statement(s) is/are: **[NSEP-2023]**

(A) the height h above the plane mirror is $h = 18.7$ cm

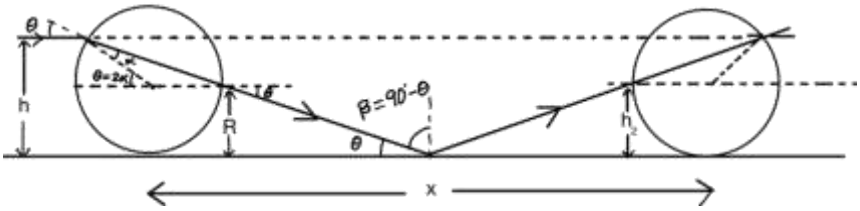
(B) the ray enters the second cylinder B at a height $h_2 = 0.1$ m

(C) the separation between the axes of the two cylinders **A** and **B** is $x = 31.54$ cm

(D) the angle of incidence on the plane mirror midway between the two cylinders is $\theta = 30^\circ$

Ans. (A, B, C, D)

Sol.



$\theta = 2\alpha$

$1\sin\theta = \mu\sin\alpha$

$1\sin 2\alpha = \sqrt{3}\sin\alpha$

$2\sin\alpha\cos\alpha = \sqrt{3}\sin\alpha$

$\cos\alpha = \dfrac{\sqrt{3}}{2}$

$\alpha = 30^\circ \Rightarrow \theta = 60^\circ$

$\beta = 90 - \theta = 30^\circ$

(A) $h = R + R\sin\theta$

$= 10 \text{ cm} + 10 \text{ cm}\sin 60^\circ$

$= 10 + 5\sqrt{3} \text{ cm} \simeq 18.7 \text{ cm}$

(B) By symmetry, $h_2 = R = 0.1 \text{ m}$

(C) $x = R + R\cot\theta + R + R\cos\theta$

$= 2R + 2R\cot 60^\circ$

$= 2\times 10 + 2\times 10\times\dfrac{1}{\sqrt{3}}$

$x = 31.54 \text{ cm}$

(D) $\beta = 30^\circ$

WAVE OPTICS

1. A slit of width a is illuminated by parallel monochromatic light of wavelength λ. The value of a at which the first minimum of the diffraction pattern will from at $\theta = 30^\circ$ is **[NSEP-2017]**

(A) $\lambda / 2$ (B) λ (C) 2λ (D) 3λ

Ans. (C)

Sol. $a\sin 30^\circ = \lambda \Rightarrow a = 2\lambda$

2. Consider the diffraction pattern due to a single slit. The first maximum for a certain monochromatic light coincides with the first minimum for red light of wavelength 660 nm . The wavelength of the monochromatic light is **[NSEP-2018]**

(A) 660 nm (B) 550 nm (C) 440 nm (D) 330 nm

Ans. (C)

Sol. $(2n+1)\dfrac{\lambda'}{2} = n\lambda$

$$\frac{3\lambda'}{2} = 660$$

$$\lambda' = \frac{660 \times 2}{3} = 440 \text{ nm}$$

3. Light of wavelength 640 nm falls on a plane diffraction grating with 12000 lines per inch. In the diffraction pattern on a screen kept at a distance of 12 cm from the grating, the distance of the second order maximum from the central maximum is **[NSEP-2019]**

(A) 1.81 cm (B) 2.41 cm (C) 3.62 cm (D) 7.25 cm

Ans. (D)

Sol. $\sin\theta = \dfrac{n\lambda}{d}$

Spacing between two consecutive slits

$$d = \frac{0.0254}{12000}$$

For second order maximum $\Rightarrow n = 2$

$$\sin\theta = \frac{2 \times 640 \times 10^{-9}}{\left(\frac{0.0254}{12000}\right)} \simeq 0.6$$

$$\frac{y}{12 \text{ cm}} = \tan\theta = 0.75 \Rightarrow y = 9 \text{ cm} \Rightarrow \text{ the closest ans is } 7.25 \text{ cm}$$

4. In many situations the point source emitting a wave starts moving, through the medium, with velocity V greater than the wave velocity in that medium. In such a case when source velocity (V) > wave velocity (V), the wave front changes **[NSEP-2019]**

(A) from spherical to plane

(B) from spherical to conical

(C) from plane to spherical

(D) from cylindrical to spherical

Ans. (B)

5. White light is used to illuminate two slits in Young's double sit experiment. Separation between the two slits is b and the screen is at a distance $D(\gg b)$ from the plane of slits. The wavelength missing at a point on the screen directly in front of one of the slits is **[NSEP-2019]**

(A) $\frac{2b^2}{3D}$ (B) $\frac{2b^2}{D}$ (C) $\frac{b^2}{3D}$ (D) $\frac{b^2}{2D}$

Ans. (C)

Sol $\Delta x = d\sin\theta \simeq d\tan\theta = dy/D$

For minima

$$\Delta x = (2n+1)\frac{\lambda}{2}$$

$$dy/D = \frac{dy}{D} = \frac{d\times d}{2D} = (2n+1)\frac{\lambda}{2}$$

$(d=b)$

$$\frac{b^2}{2D} = (2n+1)\frac{\lambda}{2}$$

$$\lambda = \frac{b^2}{(2n+1)D}\ (n=1)$$

$$\lambda = \frac{b^2}{3D}$$

6. Select the correct statement(s), out of the following, about diffraction at N parallel slits. **[NSEP-2019]**

(A) There are $(N-1)$ minima between each pair of principal maxima.
(B) There are $(N-2)$ secondary maxima between each pair of principal maxima.
(C) Width of principal maximum is proportional to $1/N$.
(D) The intensity at the principal maxima varies as N^2.

Ans. (A, B, C, D)

Sol. for N parallel slits the intensity at an angular position θ is given by

$$I = \left(A\frac{\sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta}$$

where $A =$ amplitude at slit

$$\alpha = \frac{\pi e}{\lambda}\sin\theta$$

$$\beta = \frac{\delta}{2} = (e+d)\sin\theta\left(\frac{\pi}{\lambda}\right)$$

d = opaque space between slits.
For principal maxima $B \to 0$

$$\therefore I = \left(A\frac{\sin\alpha}{\alpha}\right)^2 N^2$$

$$\therefore I \alpha N^2$$

width of principal maxima $= \frac{2\lambda}{a} = \frac{2\lambda}{N(a+e)} \alpha \frac{1}{N}$

$$a = N(d+e)$$

Case (i) : Principal maxima: the resultant amplitude will take a maximum value if $\sin\beta = 0$

$\beta = \pm n\pi;\ n = 0,1,2,3\ldots$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda \ \#(2.42)$$

$n = 0$ corresponds to zero order maximum. For $n = 1,2,3\ldots$ we obtain first second, third,.... Principal maxima respectively. The ± sign indicates that there are two principal maxima of the same order lying on either side of zero order maximum.

Case (ii) : Minima Positions : The resultant amplitude takes minimum value if $\sin N\beta = 0$ but $\sin\beta \neq 0$

$\therefore N\beta = \pm m\pi$

$$n\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$N(e+d)\sin\theta = \pm m\lambda \quad \_\_\_\_ \quad (2.43)$$

Where m has all integral values except $m = 0, N, 2N, \ldots, nN$, because for these values $\sin\beta$ becomes zero and we get principal maxima. Thus $m = 1,2,3,\ldots,(N-1)$.

Hence

$$N(e+d)\sin\theta = \pm m\lambda$$

where $m = 1,2,3\ldots(N-1),(N+1),\ldots(2N-1)\ldots$

gives the minima positions which are adjacent to the principal maxima

Case(iii) : Secondary maxima : As there are ($N-1$) minima between two adjacent principal maxima there must be ($N-2$) other maxima between two principal maxima. These are know as secondary maxima. To find their positions

$$\frac{dI}{d\beta} = 0$$

$$\frac{dI}{d\beta} = \left(A\frac{\sin\alpha}{\alpha}\right)^2 2\left(\frac{\sin N\beta}{\sin\beta}\right)\left[\frac{N\cos N\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$

$$\therefore \frac{\sin\alpha}{\alpha} \neq 0; \sin N\beta \neq 0$$

Only

$$[N\cos N\beta\sin\beta - \sin N\beta\cos\beta] = 0$$

$$N\tan\beta = \tan N\beta \quad \_\_\_\_ \quad (2.44)$$

The roots of the above equation other than those for which $\beta = \pm n\pi$ give the positions of secondary maxima.

7. The distance between two slits in Young's double slits experiment is $d = 2.5$ mm and the distance of the screen from the plane of slits is $D = 120$ cm. The slits are illuminated with coherent beam of light of wavelength $\lambda = 600$ nm. The minimum distance (from the central maximum) of a point where the intensity reduces to 25% of maximum intensity is **[NSEP-2020]**

(A) 24μ m (B) 48μ m (C) 96μ m (D) 120μ m

Ans. (C)

Sol. The resultant intensity on the screen is given by $I = I_m \cos^2\left(\frac{\pi}{\lambda} d \frac{y}{D}\right) = \frac{1}{4} I_m$ Thereby

$$\cos^2\left(\frac{\pi}{\lambda} d \frac{y}{D}\right) = \frac{1}{4} \Rightarrow \frac{\pi}{\lambda} d \frac{y}{D} = \frac{\pi}{3} \Rightarrow y = \frac{D\lambda}{3d} y = \frac{1.20 \times 600 \times 10^{-9}}{3 \times 2.5 \times 10^{-3}} = 96 \times 10^{-6} \text{ m} = 96 \mu \text{ m}$$

8. In a young's double slit experiment distance between slits is $d = 1$ mm, Wavelength of light used is 600 nm and distance of screen from the plane of slits is $D = 1$ m. the minimum distance two points on the screen where intensity falls to 75% of maximum intensity will be (Assume both sources of equal power). **[NSEP-2022]**

(A) 0.1 mm (B) 0.2 mm (C) 0.45 mm (D) 0.9 mm

Ans. (B)

Sol. $I_1 = \frac{75}{100} \times 4I_0 = 3I_0 = I_0 + I_0 + 2I_0 \cos\Delta\phi$

$\Rightarrow \cos\Delta\phi = \frac{1}{2} \Rightarrow \Delta\phi = \frac{\pi}{3}$

$\Rightarrow \frac{2\pi}{\lambda} \cdot \Delta x = \frac{\pi}{3} \Rightarrow \Delta x = \frac{\lambda}{6}$

$y = \frac{\Delta x D}{d} = \frac{\lambda D}{6d}$

Two such points will exist above and below the central maxima each at distance y from the central maxima,

$\rightarrow$ min distance $= 2y = \frac{\lambda D}{3\,d} = \frac{6 \times 10^{-7} \times 1}{3 \times 10^{-3}} = 0.2$ mm

9. Light emerges out uniformly from a point source placed at the focus of a concave mirror to give out a spherical wave front. As a result of reflection of the paraxial rays from the concave mirror, according to Huygen's theory the reflected light is in the form of a **[NSEP-2023]**

(A) spherical wave front with centre at the focus, and radius equal to the radius of curvature of the mirror

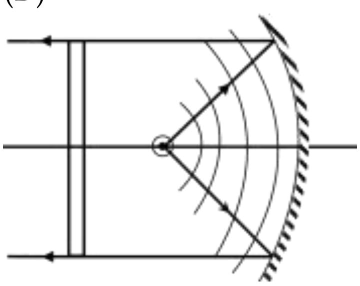
(B) spherical wave front with centre at the focus, and radius equal to the focal length of the mirror

(C) cylindrical wave front with its axis coinciding with the principal axis of the mirror.

(D) plane wave front perpendicular to the reflected beam

Ans. (D)

Sol.



Plane wave front

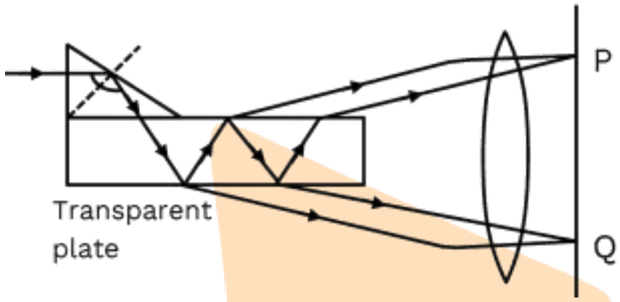
10. A parallel beam, of 6.0 mW radiation of wavelength 200 nm and of area of cross-section 1.0 mm^2, falls normally on a plane metallic surface. If the radiations are completely reflected, the pressure exerted by the radiations on the metallic surface is estimated to be

(A) 1×10^5 Pa (B) 2×10^5 Pa (C) 2×10^{-5} Pa (D) 4×10^{-5} Pa

Ans. (D)

Sol. $P_{radiation} = \frac{2I}{C} = \frac{2 \times 6 \times 10^{-3}}{3 \times 10^{8} \times 1.0 \times 10^{-6}} = 4 \times 10^{-5}$ Pa

11. In an experiment with Lummer Gehrecke plate, the two coherent beams of light, caused by multiple reflections inside the transparent plate of refractive index $\mu = 1.54$, reach the points P and Q on the screen. The net path difference between the two beams reaching either at P or Q is $\Delta x = 5000$ nm. Which of the wavelengths in the visible range ($\lambda = 390$ nm to $\lambda = 780$ nm) is / are most likely to produce a constructive interference (a maximum) at the point P as well as at Q on the screen? **[NSEP-2023]**



(A) 416.67 nm (B) 555.56 nm (C) 625.00 nm (D) 666.70 nm

Ans. (A, B, C)

Sol. 5000 nm $= n\lambda$ For maxima

$\lambda = \frac{5000}{n}$

if $n = 8, \lambda = 625$ nm

if efg $n = 9, \lambda = 555.56$ nm

if $n = 12, \lambda = 416.67$ nm